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Reconstruction of Deceleration Parameters from Recent Cosmic Observations

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In this paper, three kinds of simple parameterized deceleration parameters are reconstructed from the latest released observations of Sne Ia and Hubble data. It is found out that the transition redshift from decelerated expansion to accelerated expansion is about $z_T \sim 0.36 - 0.39$ in these three kinds of parameterizations by only using Gold Sne Ia datasets. By adding the Hubble parameter data, the z_T becomes in the range of $0.37 - 0.77$ and the errors become bigger than that in the cases of only Sne Ia datasets cases. It is also pointed out that the differences of the best fit values of transition time from decelerated expansion to accelerated expansion are due to the concrete parameterized forms. But, in 1σ range, they are consistent with each other. At last, in this way some equation of state of dark energy and some dark energy models will be rule out, but this much rely on the errors at the transition redshift.

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I. INTRODUCTION

Recent observations of High redshift Type Ia Supernova indicate that our universe is undergoing accelerated expansion which is one of the biggest challenges in present cosmological research, now [1, 2, 3, 4, 5, 6]. Meanwhile, this suggestion is strongly confirmed by the observations from WMAP [7, 8, 9, 10] and Large Scale Structure survey [11]. To understand the late-time accelerated expansion of the universe, a large part of models are proposed by assuming an extra energy component with negative pressure exists and dominates at late time to push the universe to accelerated expansion. In principle, a natural candidate for dark energy could be a small cosmological constant Λ which has the constant equation of state (EOS) $w_\Lambda = -1$. However, there exist serious theoretical problems: fine tuning and coincidence problems. To overcome the coincidence problem, the dynamic dark energy models are proposed, such as quintessence [12], phantom [13], k-essence [15], Chaplygin gas [16], holographic dark energy [17], etc., as alternative candidates.

Another approach to study the dark energy is by an almost model-independent way, i.e., the parameterized equation state of dark energy which is implemented by giving the concrete form of the equation of state of dark energy directly, such as $w(z) = w_0 + w_1 z$ [18], $w(z) = w_0 + w_1 \frac{z}{1+z}$ [19, 20], $w(z) = w_0 + w_1 \ln(1+z)$ [21], etc.. By this method, the evolution of dark dark energy with respect to the redshift z is explored, and it is found that the current constraints favore a dynamic dark energy, though the cosmological constant is not ruled out in 1σ region. But, the rapid changed dark energy, $|\frac{\partial w}{\partial z}| \ll 1$, is ruled out [22]. Also, the dark energy favors a quitom-like dark energy, i.e. crossing the cosmological constant boundary. In all, it is an effective method to rule out the dark energy models. As known, now the universe is dominated by dark energy and is undergoing accelerated expansion. However, in the past, the universe was dominated by dark matter and underwent a decelerated epoch. So, inspired by this idea, the parameterized decelerated parameter is present in almost model independent way by giving a concrete form of decelerated parameters which is positive in the past and changes into negative recently [23, 24, 25]. Moreover, it is interesting and important to know what is the transition time z_T from decelerated expansion to accelerated expansion. This is the main point of this paper to be explored. In this paper, the Sne Ia and Hubble parameter data are used to constrain the transition redshift z_T from decelerated expansion to accelerated expansion.

This paper is structured as follows. In section II, three kinds of parameterized decelerated parameter are constrained by latest 182 Sne Ia data points compiled by Riess [22] and Hubble parameter data [26]. Section III is the conclusion.

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II. RECONSTRUCTION OF DECELERATION PARAMETER

We consider a flat FRW cosmological model containing dark matter and dark energy with the metric

$$ds^2 = -dt^2 + a^2(t)dx^2. \quad (1)$$

The Friedmann equation of the flat universe is written as

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{de}), \quad (2)$$

where, $H \equiv \dot{a}/a$ is the Hubble parameter, and its derivative with respect to t is

$$\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2, \quad (3)$$

which combined with the definition of the deceleration parameter

$$q(t) = -\frac{\ddot{a}}{aH^2} \quad (4)$$

gives

$$\dot{H} = -(1+q)H^2. \quad (5)$$

By using the relation $a_0/a = 1+z$, the relation of H and q , *i.e.*, Eq. (5) can be written in its integration form

$$H(z) = H_0 \exp \left[\int_0^z [1+q(u)] d \ln(1+u) \right], \quad (6)$$

where the subscript "0" denotes the current values of the variables. If the function of $q(z)$ is given, the evolution of the Hubble parameter is obtained. In this paper, we consider three kinds of parameterized deceleration parameters:

- A. $q(z) = a + \frac{bz}{1+z}$, $H(z) = H_0 (1+z)^{1+a+b} \exp \left(-\frac{bz}{1+z} \right)$
- B. $q(z) = a + \frac{bz}{(1+z)^2}$, $H(z) = H_0 (1+z)^{1+a} \exp \left(\frac{bz}{2(1+z)^2} \right)$
- C. $q(z) = \frac{1}{2} + \frac{az+b}{(1+z)^2}$ [25], $H(z) = H_0 (1+z)^{3/2} \exp \left[\frac{b}{2} + \frac{az^2-b}{2(1+z)^2} \right]$

where, a , b are constants which can be determined from the current observations constraints. From the explicit expressions of Hubble parameters, this mechanisms can also be tried as parameterizations of Hubble parameters which can be constrained from Hubble parameter data directly.

Now, we can constrain the models by the supernovae observations and Hubble parameter data. We will use the latest released supernovae datasets and Hubble parameter data to constrain these parameterized deceleration parameters. The Gold dataset contains 182 Sne Ia data [22] by discarding all Sne Ia with $z < 0.0233$ and all Sne Ia with quality='Silver'. The 182 datasets points are used to constrain our models. Constraints from Sne Ia can be obtained by fitting the distance modulus $\mu(z)$

$$\mu_{th}(z) = 5 \log_{10}(D_L(z)) + \mathcal{M}, \quad (7)$$

where, $D_L(z)$ is the Hubble free luminosity distance $H_0 d_L(z)$ and

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')} \quad (8)$$

$$\begin{aligned} \mathcal{M} &= M + 5 \log_{10} \left(\frac{H_0^{-1}}{Mpc} \right) + 25 \\ &= M - 5 \log_{10} h + 42.38, \end{aligned} \quad (9)$$

where, M is the absolute magnitude of the object (Sne Ia). With Sne Ia datasets, the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi_{SneIa}^2(p_s) = \sum_{i=1}^N \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_i^2}, \quad (10)$$

where $N = 182$ for Gold dataset, $\mu_{obs}(z_i)$ s are the modulus obtained from observations, σ_i s are the total uncertainty of the Sne Ia data.

The best fit values for the model parameters from Hubble parameter data [26] are determined by minimizing

$$\chi_{Hub}^2(p_s) = \sum_{i=1}^9 \frac{[H_{th}(p_s; z_i) - H_{obs}(z_i)]^2}{\sigma^2(z_i)} \quad (11)$$

where H_{th} is the predicted value for the Hubble constant, H_{obs} is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty, and the summation is over the 9 Hubble parameter data points at redshifts z_i . Here, the $h_0 = 0.72$ prior is used.

Fitting the datasets from 182 Gold Sne Ia only, we obtain the minimum χ^2 s and the best fit parameters a s, b s and the transition times (redshift) z_{TS} in these three kinds of parameterizations. The results are listed in the Table I. The evolution of the decelerated parameters $q(z)$ with 1σ error are plotted in Fig. 1.

| Parameters | χ^2 | a -best | b -best | z_T |
|--|----------|-----------|-----------|------------------------|
| A. $q(z) = a + \frac{bz}{1+z}$ | 156.44 | -0.84 | 3.00 | $0.39^{+0.10}_{-0.05}$ |
| B. $q(z) = a + \frac{bz}{(1+z)^2}$ | 156.71 | -1.07 | 5.68 | $0.34^{+0.11}_{-0.05}$ |
| C. $q(z) = \frac{1}{2} + \frac{az+b}{(1+z)^2}$ | 156.54 | -1.46 | -1.46 | $0.36^{+0.12}_{-0.05}$ |

TABLE I: The best fit results the combining constraints from 182 Gold Sne Ia.

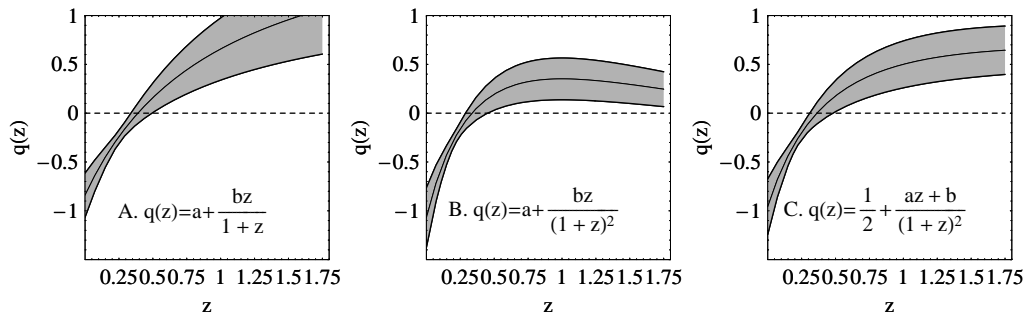


FIG. 1: The evolution of decelerated parameters with respect to the redshift z . The center solid lines is plotted with the best fit values respectively in A, B and C, where the shadows denote the 1σ regions.

The best fit parameter values of a s, b s and z_{TS} can be obtained by minimizing the combining χ^2 s of Sne Ia Gold datasets and Hubble parameter data in these three kinds of parameterizations.

$$\chi_{total}^2(p_s) = \chi_{SneIa}^2(p_s) + \chi_{Hub}^2(p_s). \quad (12)$$

The results are listed in the Table II. The evolution of the decelerated parameters $q(z)$ with 1σ error are plotted in Fig. 2.

III. DISCUSSION AND CONCLUSION

In this paper, by an almost model-independent way, we have used three kinds of parameterized decelerated parameters to obtain the transition time or redshift z_T from decelerated expansion to accelerated expansion. To obtain

| Parameters | χ^2 | a -best | b -best | z_T |
|--|----------|-----------|-----------|------------------------|
| A. $q(z) = a + \frac{bz}{1+z}$ | 162.19 | -0.66 | 1.96 | $0.51^{+0.41}_{-0.11}$ |
| B. $q(z) = a + \frac{bz}{(1+z)^2}$ | 161.01 | -0.98 | 4.99 | $0.37^{+0.21}_{-0.06}$ |
| C. $q(z) = \frac{1}{2} + \frac{az+b}{(1+z)^2}$ | 165.89 | -0.85 | -0.91 | $0.77^{+1.37}_{-0.34}$ |

TABLE II: The best fit results the combining constraints from 182 Gold Sne Ia and 9 Hubble parameter data.

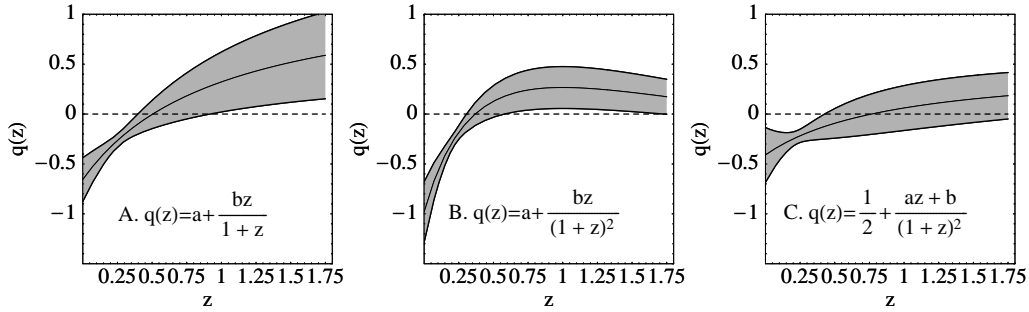


FIG. 2: The evolution of decelerated parameters with respect to the redshift z . The center solid lines is plotted with the best fit values respectively in A, B and C, where the shadows denote the 1σ regions in the combining constraints from 182 Gold Sne Ia and 9 Hubble parameter data.

the best fit values of the transition redshift, Gold Sne Ia and Hubble datasets constraints are used. Of course, the transition redshift z_T can also be derived by assuming the equation of state of dark energy. But, it will depend on the concrete forms which are designed. So, in this paper, an almost model-independent way is proposed. In this way, it is found out that the transition redshift z_T is about $z_T \sim 0.36 - 0.39$ in these three kinds of parameterizations by only using Gold Sne Ia datasets. By adding the Hubble parameter data, the z_T becomes in the range of $0.37 - 0.77$ and the errors become bigger than that in only Sne Ia cases. These may come from the uncertainty of Hubble parameter data. Also, in these three kinds of parameterized decelerated factors, the best fit values are so different, but in 1σ errors, the transition redshift is compatible with the result in Ref. [22]. Now, it is wanted to asked which one give an accurate transition redshift in the parameterized equation of state (EOS) of dark energy or parameterized decelerated parameter. The later one is advocated in this paper for its constraints from cosmic observations directly just like in the parameterized EOS of dark energy. Also, by this way, we can compare the transition redshift to rule out some dark energy models. Also, in this way, some parameterized EOS of dark energy can be tested, if the errors of transition redshift are enough small. This work will be done in the future.

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- [1] A.G. Riess, et.al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, 1998 Astron. J. 116 1009, astro-ph/9805201.
 - [2] S. Perlmutter, et.al., Measurements of omega and lambda from 42 high-redshift supernovae, 1999 Astrophys. J. 517 565, astro-ph/9812133.
 - [3] J.L. Tonry, et.al., Cosmological Results from High-z Supernovae, 2003 Astrophys. J. 594 1, astro-ph/0305008;
 - [4] R.A. Knop, et.al., New Constraints on Ω_M , Ω_Λ , and w from an Independent Set of Eleven High-Redshift Supernovae Observed with HST, astro-ph/0309368.
 - [5] B.J. Barris, et.al., 23 High Redshift Supernovae from the IfA Deep Survey: Doubling the SN Sample at $z > 0.7$, 2004 Astrophys.J. 602 571, astro-ph/0310843.

- [6] A.G. Riess, et.al., Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, astro-ph/0402512.
- [7] P. de Bernardis, et.al., A Flat Universe from High-Resolution Maps of the Cosmic Microwave Background Radiation, 2000 Nature 404 955, astro-ph/0004404
- [8] S. Hanany, et.al., MAXIMA-1: A Measurement of the Cosmic Microwave Background Anisotropy on angular scales of 10 arcminutes to 5 degrees, 2000 Astrophys. J. 545 L5, astro-ph/0005123.
- [9] D.N. Spergel et.al., First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, 2003 Astrophys. J. Supp. 148 175, astro-ph/0302209.
- [10] D.N. Spergel et al 2006, astro-ph/0603449.
- [11] M. Tegmark et al., Phys. Rev. D 69 (2004) 103501, astro-ph/0310723; M. Tegmark et al., Astrophys. J. 606 (2004) 702, astro-ph/0310725.
- [12] I. Zlatev, L. Wang, and P.J. Steinhardt, Quintessence, Cosmic Coincidence, and the Cosmological Constant, 1999 Phys. Rev. Lett. 82 896, astro-ph/9807002; P.J. Steinhardt, L. Wang, I. Zlatev, Cosmological Tracking Solutions, 1999 Phys. Rev. D 59 123504, astro-ph/9812313; M.S. Turner, Making Sense Of The New Cosmology, 2002 Int. J. Mod. Phys. A 17S1 180, astro-ph/0202008; V. Sahni, The Cosmological Constant Problem and Quintessence, 2002, Class.Quant.Grav. 19 3435, astro-ph/0202076.
- [13] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phantom Energy: Dark Energy with $w < -1$ Causes a Cosmic Doomsday, 2003 Phys. Rev. Lett. 91 071301, astro-ph/0302506; R.R. Caldwell, A Phantom Menace? Cosmological consequences of a dark energy component with super-negative equation of state, 2002 Phys. Lett. B 545 23, astro-ph/9908168; P. Singh, M. Sami, N. Dadhich, Cosmological dynamics of a phantom field, 2003 Phys. Rev. D 68 023522, hep-th/0305110; J.G. Hao, X.Z. Li, Attractor Solution of Phantom Field, 2003 Phys. Rev. D 67 107303, gr-qc/0302100.
- [14] Feng B et al 2005 Phys. Lett. B 607(1-2) 35.
- [15] Armendariz-Picon, T. Damour, V. Mukhanov, k-Inflation, 1999 Physics Letters B 458 209; M. Malquarti, E.J. Copeland, A.R. Liddle, M. Trodden, A new view of k-essence, 2003 Phys. Rev. D 67 123503; T. Chiba, Tracking k-essence, 2002 Phys. Rev. D 66 063514, astro-ph/0206298.
- [16] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B 511 (2001) 265, gr-qc/0103004; N. Bilic, G. B. Tupper, and R. D. Viollier, Phys. Lett. B 535 (2002) 17 [astro-ph/0111325]; M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D 66 (2002) 043507, gr-qc/0202064.
- [17] M. Li, Phys. Lett. B 603 (2004) 1, hep-th/0403127; K. Ke and M. Li, Phys. Lett. B 606 (2005) 173, hep-th/0407056; Y. Gong, Phys. Rev. D 70 (2004) 064029, hep-th/0404030; Y. S. Myung, Phys. Lett. B 610 (2005) 18, hep-th/0412224; Q. G. Huang and M. Li, JCAP 0408 (2004) 013, astro-ph/0404229; Q. G. Huang and M. Li, JCAP 0503 (2005) 001, hep-th/0410095; Q. G. Huang and Y. Gong, JCAP 0408 (2004) 006, astro-ph/0403590; Y. Gong, B. Wang and Y. Z. Zhang, Phys. Rev. D 72 (2005) 043510, hep-th/0412218; Z. Chang, F.-Q. Wu, and X. Zhang, astro-ph/0509531.
- [18] A.R. Cooray and D. Huterer, *Astrophys. J.* **513** L95 (1999).
- [19] M. Chevallier, D. Polarski, *Int. J. Mod. Phys. D* **10** 213 (2001); gr-qc/0009008.
- [20] E.V. Linder, *Phys. Rev. Lett.* **90** 091301 (2003).
- [21] B.F. Gerke and G. Efstathiou, *Mon. Not. Roy. Astron. Soc.* **335** 33 (2002).
- [22] A.G. Riess et al., astro-ph/0611572.
- [23] N. Banerjee, S. Das, *Acceleration of the universe with a simple trigonometric potential*, astro-ph/0505121.
- [24] L. Xu, H. Liu and Y. Ping, *Reconstruction of Five-dimensional Bounce cosmological Models From Deceleration Factor*, Int. Jour. Theor. Phys. **45**, 869-876, (2006), astro-ph/0601471.
- [25] Y. Gong, A. Wang, *Reconstruction of the deceleration parameter and the equation of state of dark energy*, astro-ph/0612196.
- [26] Simon, J., Verde, L., & Jimenez, R. 2005, Phys. Rev. D, 71, 123001, astro-ph/0412269.